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A Stable Inverting Integrator with an Extended High-Frequency Range

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***Abstract*—A stable inverting integrator is described. The integrator is obtained by inverting the transfer function of a passive *RC* differentiator. The integrator uses one operational amplifier. The resulting integrator has a builtin zero that could extend its high-frequency range of operation considerably beyond that of the Miller integrator. Experimental and simulation results verify the feasibility of the basic concept and the proposed circuit.**

***Index Terms*—Active circuits, analog signal processing, circuit design, filters, integrators, operational amplifiers, transfer functions.**

I. Introduction

The inverting Miller integrator is limited at high frequencies to about one-tenth of the unity gain bandwidth of the amplifier. Poles at frequencies equal to or greater than the unity-gain bandwidth are the culprits [1]. A great deal of research efforts were employed in attempting to extend the high-frequency range through passive and active compensation schemes [1]–[6].

One method of passive compensation introduces an additional amount of phase lead by adding a resistor in series with the capacitor of the Miller integrator. Another method shunts the resistance of the Miller integrator with a capacitor [3, pp. 183–184], [4, pp. 232–241], [5, pp. 204–206, 220, 223–226]. Both methods depend on pole-zero cancellation. However, two obvious disadvantages of these methods are stated in [4] and [5]. The first disadvantage is that the unity gain bandwidth varies among operational amplifiers, and thus each amplifier should be compensated individually. In [3] the use of a potentiometer as a series resistance is suggested to overcome this disadvantage. The second disadvantage is that under changing ambient conditions, as explained in [3]–[5], the compensation will no longer be satisfactory because they attempt to "match two electrically dissimilar elements to each other" [5, p. 220]. Active compensation is employed to alleviate the disadvantages of passive compensation by utilizing "matching" operational amplifiers [3]–[5]. A disadvantage of the active compensation is that it would not work adequately if the operational amplifiers are not properly matched.

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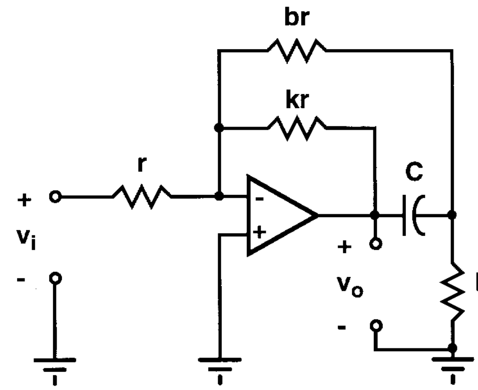


Fig. 1. The proposed inverting integrator.

The approach in this brief extends the high-frequency range of the integrator without having the disadvantages of either the passive or the active compensation techniques. Indeed, at high-frequency, the frequency range is limited, as will be shown, by the relation $\omega \ll 1/RC$ rad/s. Thus, by choosing $1/RC \gg \omega_c$, where ω_c is the unity gain bandwidth of the operational amplifier, we obtain an integrator limited by the higher dynamics of the operational amplifier and requiring neither the active compensation of "matching" operational amplifiers nor the passive compensation of "matching dissimilar elements."

The approach presented in this brief did not evolve from the view of compensating the Miller integrator, but rather from the idea of obtaining an integrator by inverting the traditional passive RC differentiator. In [6] the traditional approach to inverse system design was employed to design a noninverting integrator. In this brief, the approach is used to develop an inverting integrator that extends the high-frequency range considerably beyond that of the Miller integrator. The resulting integrator has a builtin zero that could be used to control the frequency range of the integrator to achieve integrators with appropriately lower or higher frequency ranges than is possible by using the traditional integrators. Note that the resulting integrator frequency range will thus be limited at the upper end by the inverse of the RC time constant, i.e., $\omega \ll 1/RC$ rad/s, while for the traditional Miller integrator it is the lower frequency end that will be limited by the inverse of the RC time constant, i.e., $\omega \gg 1/A_oRC$, where A_o is the dc gain of the operational amplifier. Indeed, very low-frequency integrators could easily be obtained by using the new approach. In addition, the resulting circuit acts as an amplifier for dc input voltages and thus exhibits a builtin low-frequency stability. At high frequencies, it could be shown that the resulting transfer function could be obtained by a pole-zero cancellation. However, both the pole and the zero occur at a radian frequency of $1/RC$, which is quite different than trying to cancel out a pole at the unity gain bandwidth frequency of the operational amplifier as the passive and active compensation techniques aspire to do. The usable high-frequency range of the integrator approaches the unity gain bandwidth of the operational amplifier.

TABLE I
THE FREQUENCY RANGES AND THE CORRESPONDING TRANSFER FUNCTIONS OF THE NEW INVERTING INTEGRATOR

	Frequency Range	Assumptions	Transfer Function
Low-Frequency	$\frac{b}{kRC} \ll \omega \ll \frac{1}{RC}$	$\lvert A(s)\rvert \gg 1;\ \frac{b+1}{b\lvert A(s)\rvert} \ll \frac{1}{k}$	$\frac{V_o(s)}{V_i(s)} \approx \frac{-kb}{b+kRCs}$
Mid-Frequency	$\frac{(b+kRC)\omega_c}{k(b+1)} \ll \omega \ll \frac{1}{RC}$	$\lvert A(s)\rvert \gg 1;\ A(s) \approx \frac{\omega_c}{s}$	$\frac{V_o(s)}{V_i(s)} \approx \frac{-kb\omega_c}{k(b+1)s+(b+kRC)\omega_c}$
High-Frequency	$\frac{b\omega_c}{k(b+1)} \ll \omega \ll \frac{1}{RC}$	$RC\,\omega_c \ll 1;\ A(s) \approx \frac{\omega_c}{s}$	$\frac{V_o(s)}{V_i(s)} \approx \frac{-kb\omega_c}{b\omega_c+k(b+1)s}$

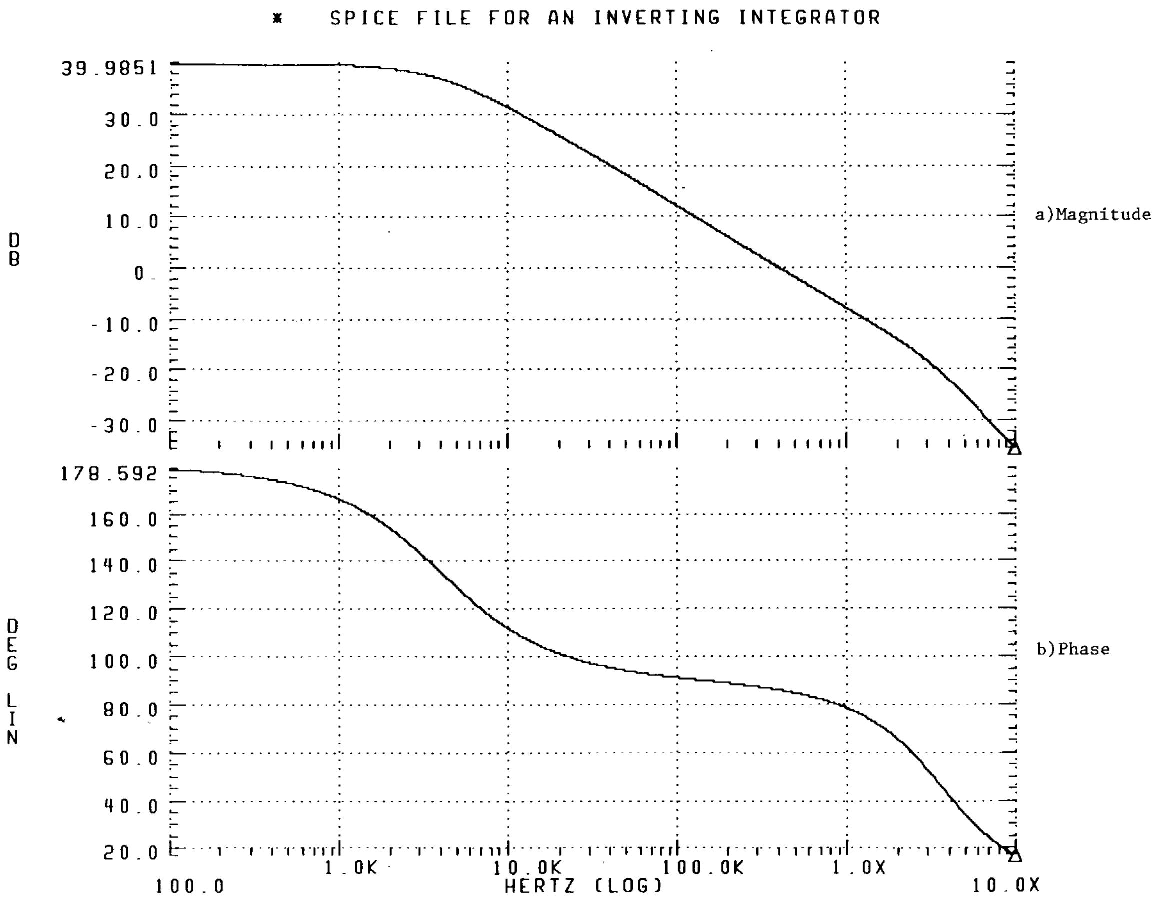


Fig. 2. HSPICE simulation of the integrator with the values of $r = 10\ \text{k}\Omega, b = 0.5, k = 100, R = 500\ \Omega$, and $C = 100$ pF. (a) The magnitude of the voltage transfer function, in decibels. (b) The phase of the voltage transfer function in degrees.

In [6], the addition of a second operational amplifier, to be used as a buffer, was suggested to avoid loading. The buffer may be dispensed with by proper loading. The proposed inverting integrator dispenses with the buffer for high-frequency applications, by the proper choice of component values to avoid loading, and thus a single amplifier circuit is obtained. For a low-frequency integrator, a buffer may be used since its dynamics does not influence the operation of the integrator at low frequencies.

II. THE NEW INVERTING INTEGRATOR

The new inverting integrator is shown in Fig. 1. Nodal analysis of Fig. 1, similar to the analysis carried out in [6], yields

$$\frac{V_o(s)}{V_i(s)} \approx \frac{-1}{\dfrac{A(s)+1}{kA(s)} + \dfrac{b+1}{bA(s)} + \dfrac{RCs - \dfrac{R}{brA(s)}}{b(1+RCs)+\dfrac{R}{r}}}. \tag{1}$$

Assuming that $R \ll br, k \gg b$, and $\omega \gg 1/brC|A(s)|$, (1) simplifies to

$$\frac{V_o(s)}{V_i(s)} \approx \frac{-1}{\dfrac{1}{kA(s)} + \dfrac{b+1}{bA(s)} + \dfrac{b+kRCs}{kb(1+RCs)}}. \tag{2}$$

Note that the RC circuit should always be chosen so that it would act as a passive differentiator in the frequency range of interest, i.e., [6]

$$\omega \ll \frac{1}{RC}. \tag{3}$$

Equation (3) implies that $|RCs| \ll 1$ and that the RC product should be chosen appropriately for the frequency range of interest. Thus the resistance and capacitor should be chosen to give large RC product values for low-frequency integrators and should be chosen to give small RC product values for high-frequency integrators.

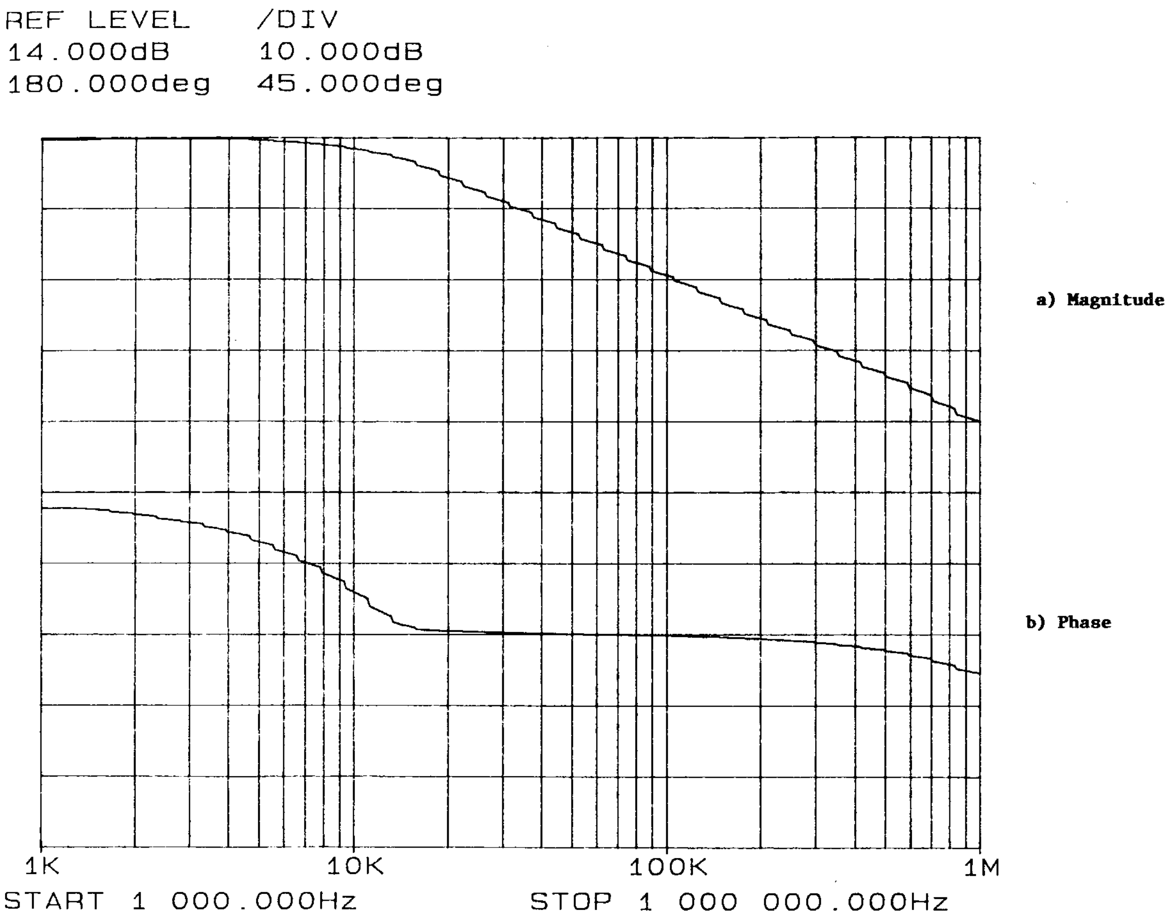


Fig. 3. Experimental results of the integrator using the same values used for the simulation. (a) The magnitude of the voltage transfer function in decibels. (b) The phase of the voltage transfer function in degrees.

Utilizing (3), (2) simplifies to

$$\frac{V_o(s)}{V_i(s)} \approx \frac{-1}{\dfrac{1}{kA(s)} + \dfrac{b+1}{bA(s)} + \dfrac{1}{k} + \dfrac{RCs}{b}}. \tag{4}$$

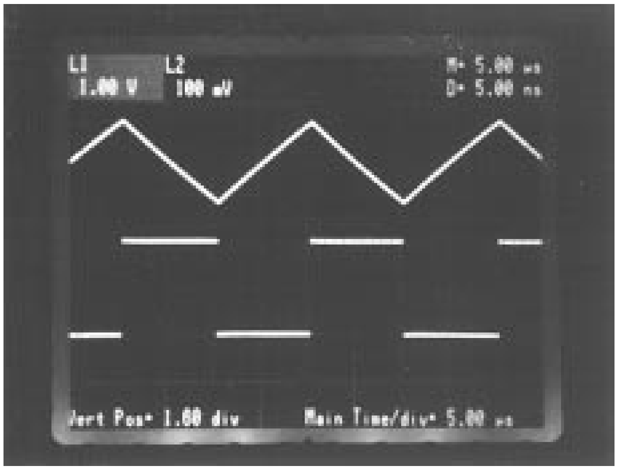
Table I shows the resulting transfer functions, under the shown assumptions, for the low-, the mid-, and the high-frequency ranges. Note that for the low- and mid-frequency ranges, the frequency is less than one tenth of the unity gain bandwidth. The transfer functions are obtained by applying the respective assumptions to (4). The lower limits of the frequency ranges are obtained from the corresponding transfer functions and the upper limits are obtained from (3). Note that for both the mid- and high-frequency ranges, $A(s)$ is approximated by a one-pole model such that at high enough frequencies it can be expressed as $A(s) \approx \omega_c/s$. An alternative approach to obtaining the transfer function for the high-frequency range is to substitute $A(s) \approx \omega_c/s$ in (2) and choose $RC\omega_c \ll 1$. The denominator could then be factored to approximate a pole-zero cancellation at the radian frequency $1/RC$. The resulting transfer function is the same as that shown in Table I, and the lower frequency limit is a function of the operational amplifier unity gain bandwidth in a manner reminiscent of active R integrators. However, in contrast to active R integrators, the lower limit of the frequency range is controlled by the designer. In addition, the upper limit could be made greater than that possible in active R integrators. For example, the integrating summer reported in [8] yields a transfer function with a pole that limits the upper frequency range of the integrators. It is instructive to compute the quality factor, Q, of the new integrator for the high-frequency region [5], [6]. If the transfer function of an integrator is expressed as $T(j\omega) = 1/[R(\omega) + jX(\omega)]$, then the integrator Q-factor is defined as $Q = X(\omega)/R(\omega)$. From the transfer function shown in Table I for the high-frequency region, we obtain $Q = k(b+1)\omega/b\omega = k(b+1)/b|A|$. Thus, Q is no longer totally dependent on the operational amplifier open-loop characteristics, and second, Q is an increasing function of frequency. Thus for $b = 0.5, k = 100$, and $\omega = \omega_c/3$, we obtain the value $Q = 100$. The corresponding Q value for the Miller integrator is $Q = -|A| = -3$, while for the actively compensated Miller integrator [5], we obtain $Q = -|A|^3 = -9$.

III. Stability Proof

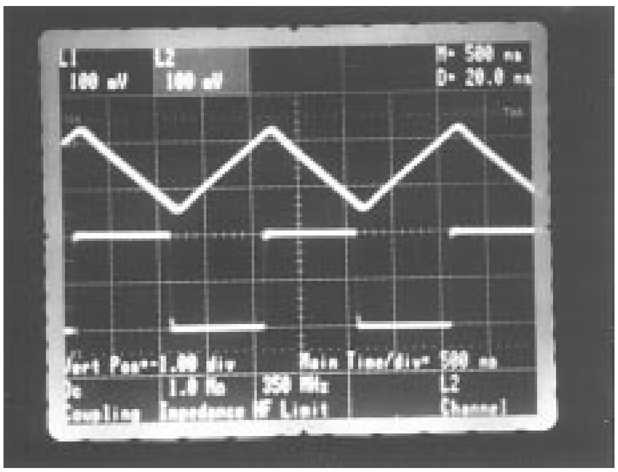
Following the approach of Martin and Sedra [2], a two-pole model is used for the operational amplifier. Thus substituting $A(s) = \omega_c\omega_2/s(s+\omega_2)$ in (1) yields

$$\begin{aligned}\frac{V_o(s)}{V_i(s)} &\approx \frac{-1}{\dfrac{(b+1)(\omega_2+s)s}{b\omega_c\omega_2} + \dfrac{1}{k} + \dfrac{s(s+\omega_2)}{k\omega_c\omega_2} + \dfrac{RCs}{b(1+RCs)}}\\ &= \frac{-kb\omega_c\omega_2(1+RCs)}{a_3s^3 + a_2s^2 + a_1s + a_o}.\end{aligned} \tag{5}$$

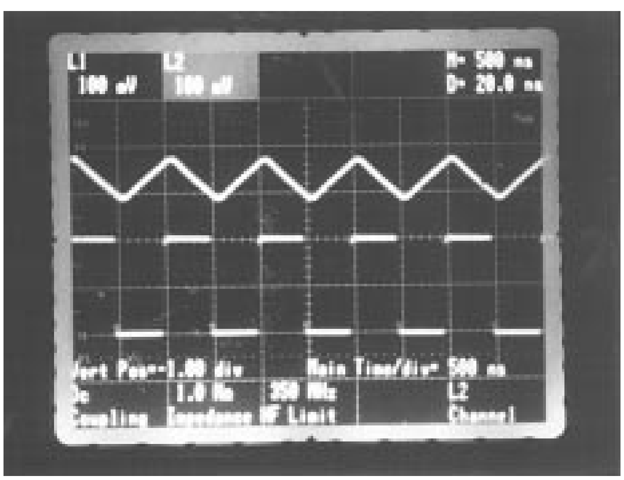
For a third-order system to be stable, using the Routh–Hurwitz stability criterion, it is necessary and sufficient that the coefficients be of the same sign and satisfy the inequality $a_2a_1 > a_oa_3$ [7]. It is easy to check that this inequality is satisfied and that the system is stable.



(a)



(b)



(c)

Fig. 4. The triangular waveform responses of the circuit to square waveform inputs at the frequencies (a) 50 kHz, (b) 500 kHz, and (c) 1 MHz.

IV. Simulation and Experimental Results

The circuit of Fig. 1 was simulated using HSPICE with $r = 10\ \text{k}\Omega, k = 100, b = 0.5, R = 500\ \Omega, C = 100$ pF, and LM741 (National Semiconductor) for the operational amplifier. The simulation results for the magnitude and phase are shown in Fig. 2. The simulation shows that the circuit acts as an integrator with $\pm 10\%$ phase error from about 22 kHz to over 700 kHz. The corresponding experimental results for the magnitude and phase, using HP3577A Network Analyzer, are shown in Fig. 3 and agree overall with the simulation results.

The discrepancy between the simulation and experimental results is due to the fact that the simulation did not take into consideration stray capacitors. Additionally the HP3577A might have needed some tuning. The extended frequency range of the integrator is demonstrated by the triangular wave responses to the square wave inputs at frequencies of 50 kHz, 500 kHz, and 1 MHz as shown in Fig. 4(a), (b), and (c), respectively.

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